ANALOG DEVICES

Mini Tutorial
MT-206

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The Chebyshev Response

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IN THIS MINI TUTORIAL

The Chebyshev filter, a circuit incorporating precision operational amplifiers (op amps), is one of the discrete circuits described in a series of mini tutorials.

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REVISION HISTORY

1/12—Revision 0: Initial Version

INTRODUCTION TO THE CHEBYSHEV RESPONSE

The Chebyshev (or Chevyshev, Tschebychev, Tschebyscheff, or Tchevysheff, depending on how one translates from Russian) filter has a smaller transition region than the same-order Butterworth filter, at the expense of ripples in its pass-band and slightly degraded transient response. This filter gets its name because the Chebyshev filter minimizes the height of the maximum ripple, which is the Chebyshev criterion.

Chebyshev filters have 0 dB relative attenuation at dc. Odd-order filters have an attenuation band that extends from 0 dB to the ripple value. Even-order filters have a gain equal to the pass-band ripple. The number of cycles of ripple in the pass band is equal to the order of the filter.

The poles of the Chebyshev filter can be determined by moving the poles of the Butterworth filter to the right (towards the axis), forming an ellipse. This is accomplished by multiplying the real part of the pole by k_r and the imaginary part by k_I. The values k_r and k_I can be computed by

$$K_r = \sinh A \quad (1)$$

$$K_I = \cosh A \quad (2)$$

where:

$$A = \frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon} \quad (3)$$

where n is the filter order and

$$\varepsilon = \sqrt{10^R - 1} \quad (4)$$

$$R = \frac{R_{dB}}{10} \quad (5)$$

where:

$$R_{dB} = \text{pass-band ripple in dB} \quad (6)$$

Figure 1 shows the pole locations for a 5-pole 1 dB Chebyshev filter.

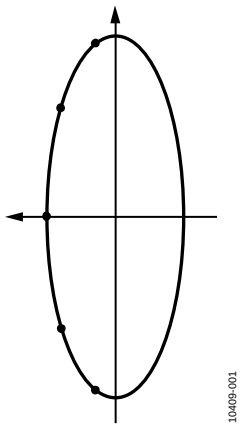

Figure 1. 1 dB Chebyshev Pole Location

The Chebyshev filters are typically normalized so that the edge of the ripple band is at ωo = 1.

The 3 dB bandwidth is given by

$$A_{3dB} = \frac{1}{n}\cosh^{-1}\left(\frac{1}{\varepsilon}\right) \quad (7)$$

This is tabulated in Table 1.

Table 1. 3 dB Bandwidth to Ripple Bandwidth

Order	0.01 dB	0.1 dB	0.25 dB	0.5 dB	1 dB
2	3.30362	1.93432	1.59814	1.38974	1.21763
3	1.87718	1.38899	1.25289	1.16749	1.09487
4	1.46690	1.21310	1.13977	1.09310	1.05300
5	1.29122	1.13472	1.08872	1.05926	1.03381
6	1.19941	1.09293	1.06134	1.04103	1.02344
7	1.14527	1.06800	1.04495	1.03009	1.01721
8	1.11061	1.05193	1.03435	1.02301	1.01316
9	1.08706	1.04095	1.02711	1.01817	1.01040
10	1.07033	1.03313	1.02194	1.01471	1.00842

FREQUENCY RESPONSE, GROUP DELAY, IMPULSE RESPONSE, AND STEP RESPONSE

The frequency response, group delay, impulse response, and step response, as well as the amplitude, are cataloged in Figure 2 through Figure 26 for various values of pass-band ripple (0 .01 dB, 0.1 dB, 0.25 dB, 0.5 dB, and 1 dB).

0.01 DB CHEBYSHEV RESPONSE

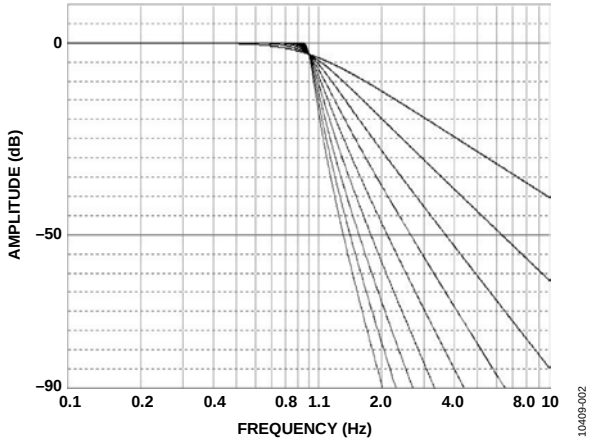

Figure 2. 0.01 dB Chebyshev Response, Amplitude

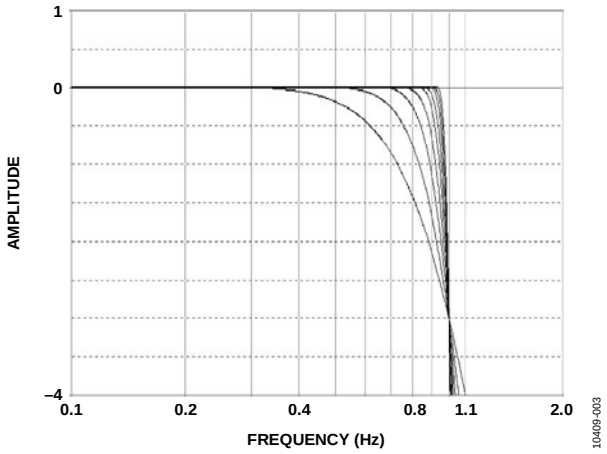

Figure 3. 0.01 dB Chebyshev Response, Amplitude Detail

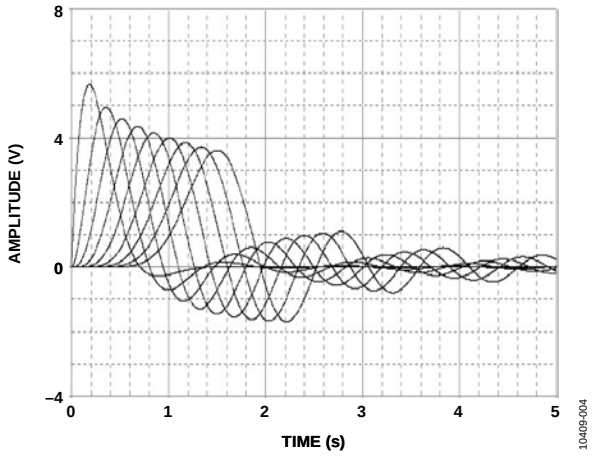

Figure 4. 0.01 dB Chebyshev Response, Impulse Response

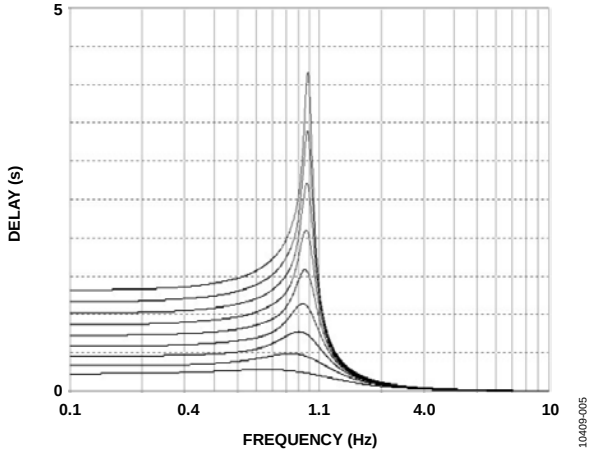

Figure 5. 0.01 dB Chebyshev Response, Group Response

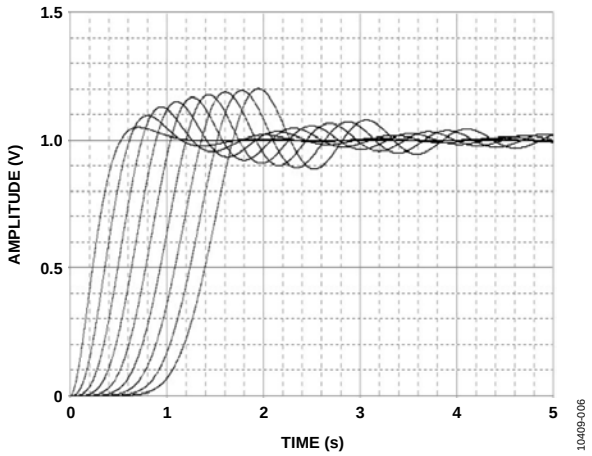

Figure 6. 0.01 dB Chebyshev Response, Step Response

0.1 DB CHEBYSHEV RESPONSE

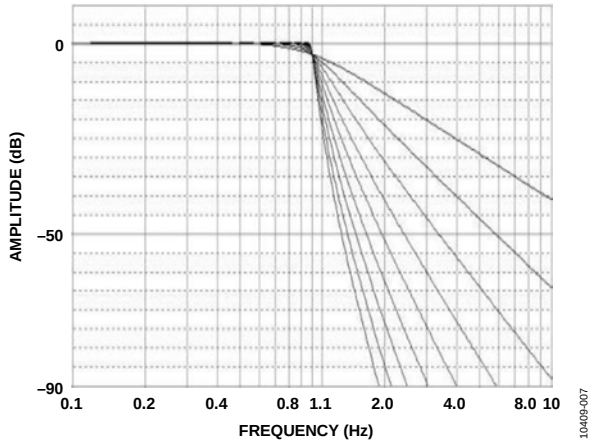

Figure 7. 0.1 dB Chebyshev Response, Amplitude

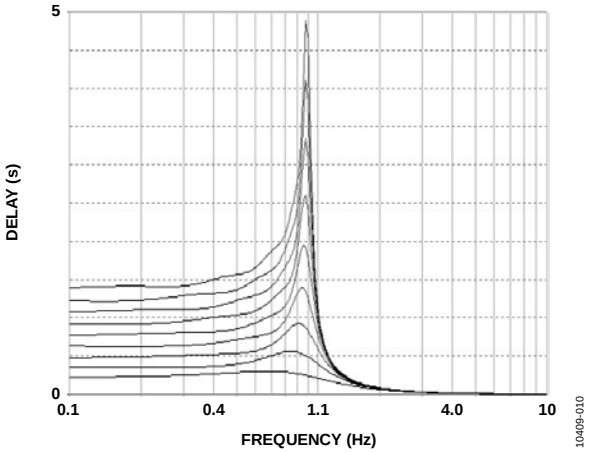

Figure 10. 0.1 dB Chebyshev Response, Group Delay

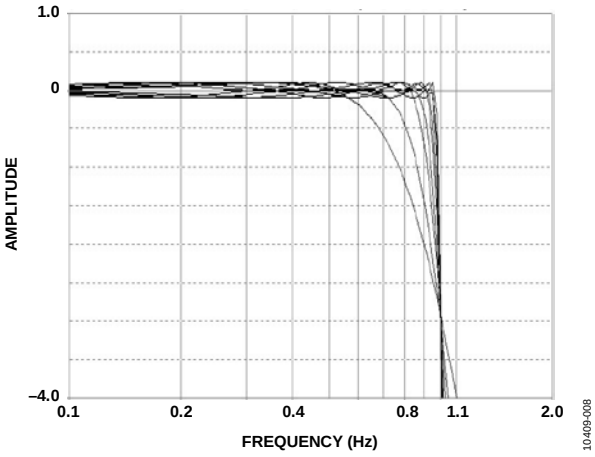

Figure 8. 0.1 dB Chebyshev Response, Amplitude Detail

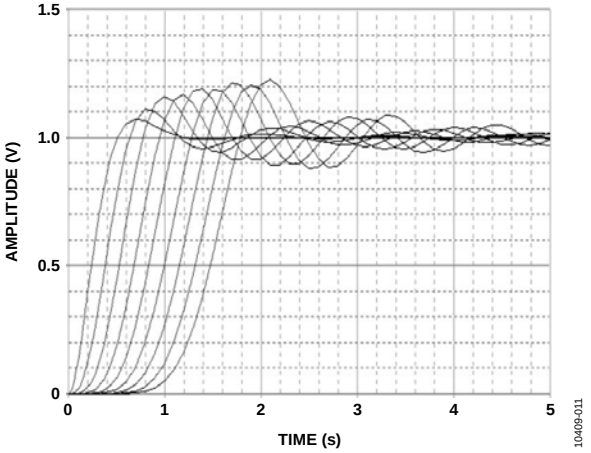

Figure 11. 0.1 dB Chebyshev Response, Step Response

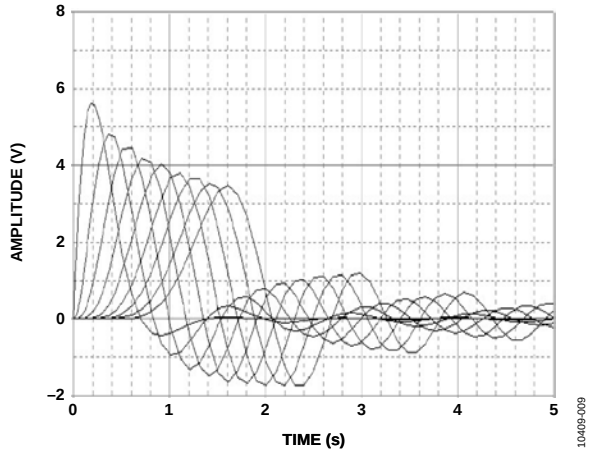

Figure 9. 0.1 dB Chebyshev Response, Impulse Response

0.25 DB CHEBYSHEV RESPONSE

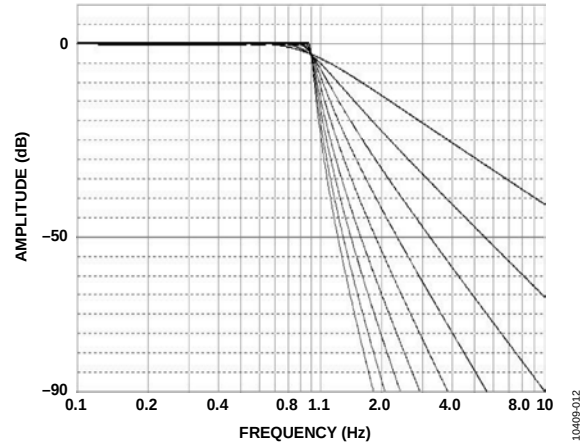

Figure 12. 0.25 dB Chebyshev Response, Amplitude

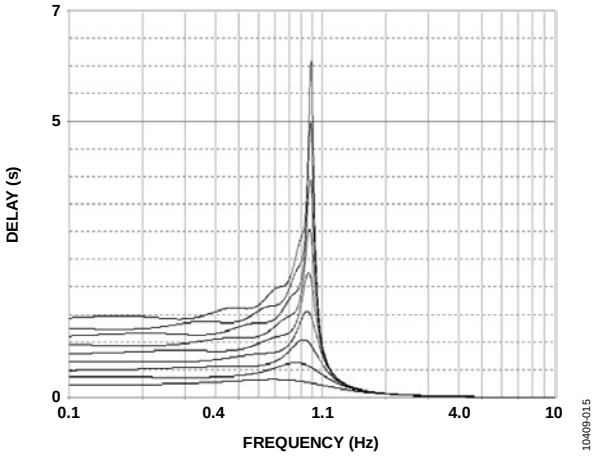

Figure 15. 0.25 dB Chebyshev Response, Group Delay

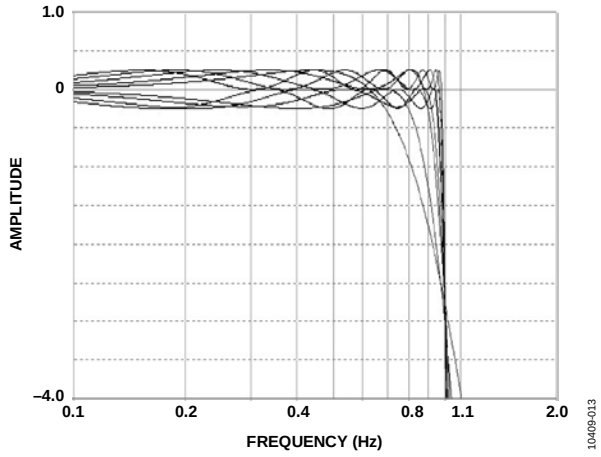

Figure 13. 0.25 dB Chebyshev Response, Amplitude Detail

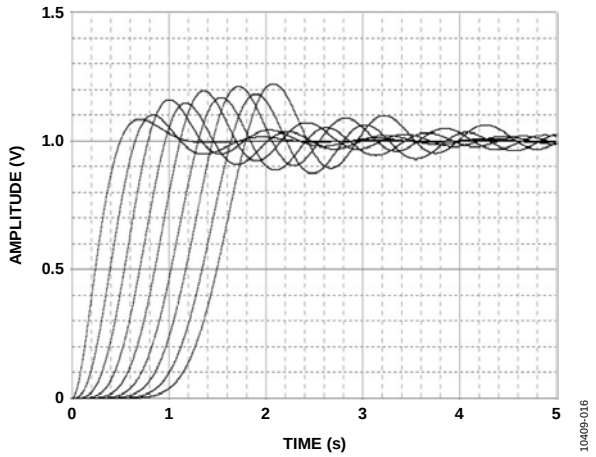

Figure 16. 0.25 dB Chebyshev Response, Step Response

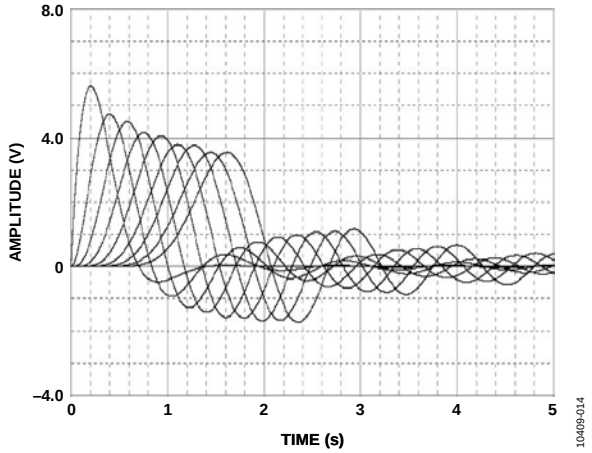

Figure 14. 0.25 dB Chebyshev Response, Impulse Response

0.5 DB CHEBYSHEV RESPONSE

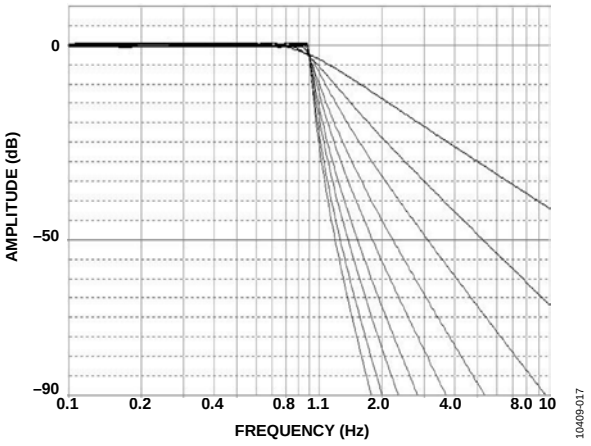

Figure 17. 0.5 dB Chebyshev Response, Amplitude

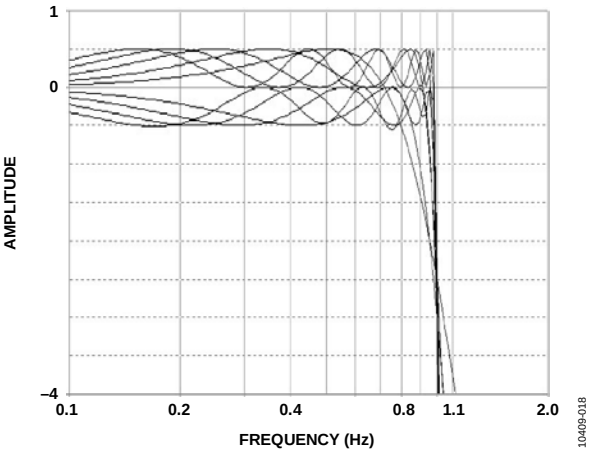

Figure 18. 0.5 dB Chebyshev Response, Amplitude Detail

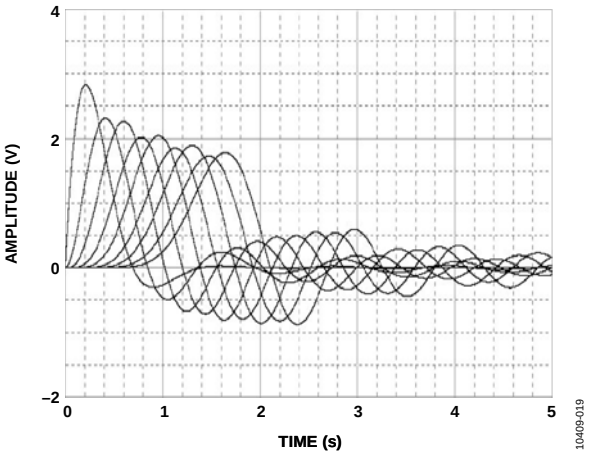

Figure 19. 0.5 dB Chebyshev Response, Impulse Response

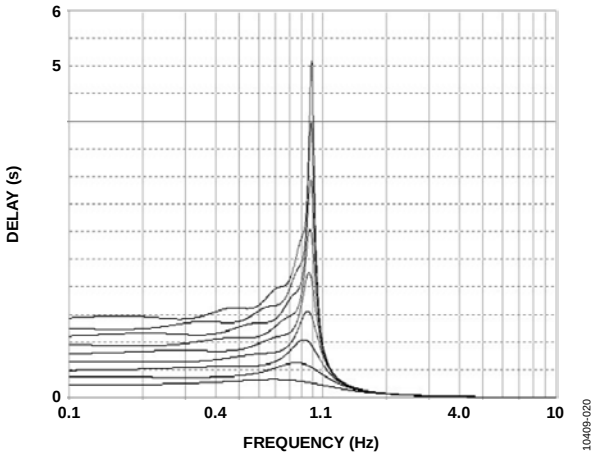

Figure 20. 0.5 dB Chebyshev Response, Group Delay

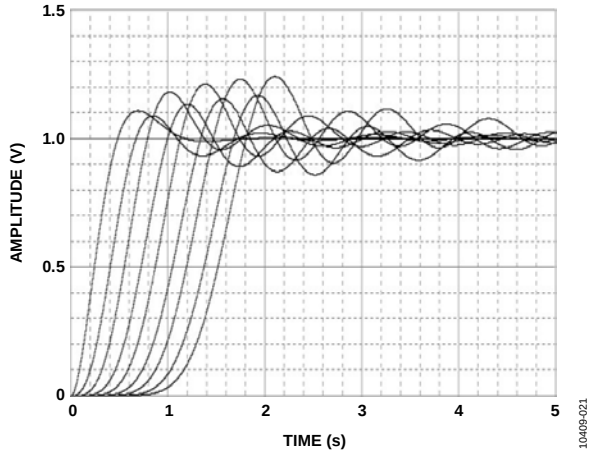

Figure 21. 0.5 dB Chebyshev Response, Step Response

1 DB CHEBYSHEV RESPONSE

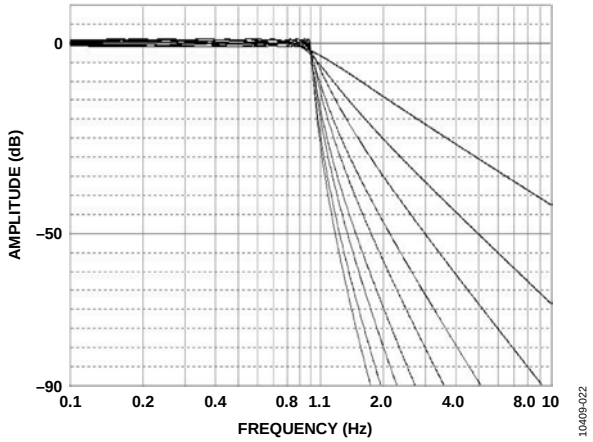

Figure 22. 1 dB Chebyshev Response, Amplitude

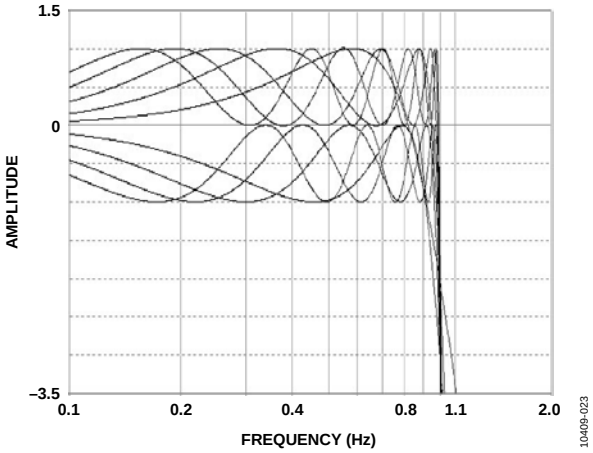

Figure 23. 1 dB Chebyshev Response, Amplitude Detail

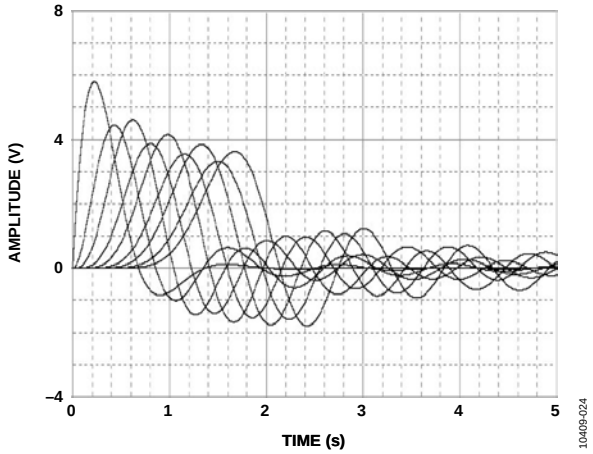

Figure 24. 1 dB Chebyshev Response, Impulse Response

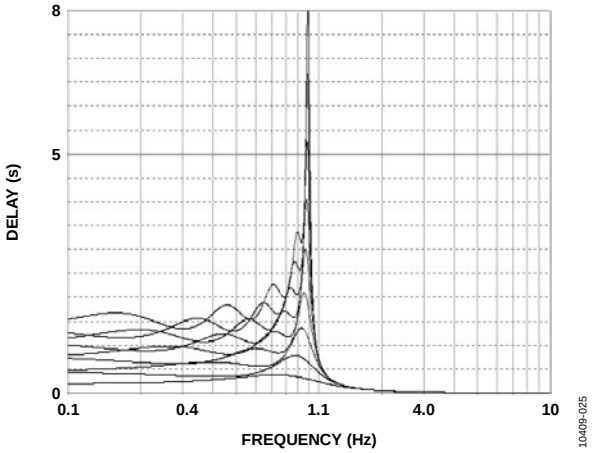

Figure 25. 1 dB Chebyshev Response, Group Delay

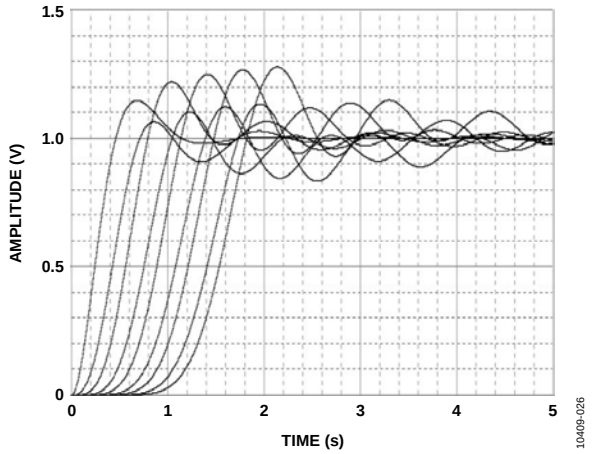

Figure 26. 1 dB Chebyshev Response, Step Response

POLE LOCATION TABLES

The pole locations and corresponding ω_o and α terms for these values of ripple are tabulated in Table 2 through Table 6.

0.01 DB CHEBYSHEV DESIGN

Table 2.

Order	Section	Real Part	Imaginary Part	F_0	α	Q	−3 db Frequency	Peaking Frequency	Peaking Level
2	1	0.6743	0.7075	0.9774	1.3798	0.7247		0.2142	0.0100
3	1	0.4233	0.8663	0.9642	0.8780	1.1389		0.7558	2.0595
	2	0.8467		0.8467			0.8467		
4	1	0.6762	0.3828	0.7770	1.7405	0.5746	0.6069		
	2	0.2801	0.9241	0.9656	0.5801	1.7237		0.8806	5.1110
5	1	0.5120	0.5879	0.7796	1.3135	0.7613		0.2889	0.0827
	2	0.1956	0.9512	0.9711	0.4028	2.4824		0.9309	8.0772
	3	0.6328		0.6328			0.6328		
6	1	0.5335	0.2588	0.5930	1.7995	0.5557	0.4425		
	2	0.3906	0.7072	0.8079	0.9670	1.0342		0.5895	1.4482
	3	0.1430	0.9660	0.9765	0.2929	3.4144		0.9554	10.7605
7	1	0.4393	0.4339	0.6175	1.4229	0.7028	0.6136		
	2	0.3040	0.7819	0.8389	0.7247	1.3798		0.7204	3.4077
	3	0.1085	0.9750	0.9810	0.2212	4.5208		0.9689	13.1578
	4	0.4876		0.4876			0.4876		
8	1	0.4268	0.1951	0.4693	1.8190	0.5498	0.3451		
	2	0.3168	0.5556	0.6396	0.9907	1.0094		0.4564	1.3041
	3	0.2418	0.8315	0.8659	0.5585	1.7906		0.7956	5.4126
	4	0.0849	0.9808	0.9845	0.1725	5.7978		0.9771	15.2977
9	1	0.3686	0.3420	0.5028	1.4661	0.6821	0.4844		
	2	0.3005	0.6428	0.7096	0.8470	1.1807		0.5682	2.3008
	3	0.1961	0.8661	0.8880	0.4417	2.2642		0.8436	7.3155
	4	0.0681	0.9848	0.9872	0.1380	7.2478		0.9824	17.2249
	5	0.3923		0.3923			0.3923		
10	1	0.3522	0.1564	0.3854	1.8279	0.5471	0.2814		
	2	0.3178	0.454	0.5542	1.1469	0.8719		0.3242	0.5412
	3	0.2522	0.7071	0.7507	0.6719	1.4884		0.6606	3.9742
	4	0.1619	0.891	0.9056	0.3576	2.7968		0.8762	9.0742
	5	0.0558	0.9877	0.9893	0.1128	8.8645		0.9861	18.9669

0.1 DB CHEBYSHEV DESIGN

Table 3.

Order	Section	Real Part	Imaginary Part	F_0	α	Q	−3 db Frequency	Peaking Frequency	Peaking Level
2	1	0.6104	0.7106	0.9368	1.3032	0.7673		0.3638	0.0999
3	1	0.3490	0.8684	0.9359	0.7458	1.3408		0.7952	3.1978
	2	0.6970		0.6970			0.6970		
4	1	0.2177	0.9254	0.9507	0.4580	2.1834		0.8994	7.0167
	2	0.5257	0.3833	0.6506	1.6160	0.6188	0.5596		
5	1	0.3842	0.5884	0.7027	1.0935	0.9145		0.4457	0.7662
	2	0.1468	0.9521	0.9634	0.3048	3.2812		0.9407	10.4226
	3	0.4749		0.4749			0.4749		
6	1	0.3916	0.2590	0.4695	1.6682	0.5995	0.3879		
	2	0.2867	0.7077	0.7636	0.7509	1.3316		0.6470	3.1478
	3	0.1049	0.9667	0.9724	0.2158	4.6348		0.9610	13.3714
7	1	0.3178	0.4341	0.5380	1.1814	0.8464		0.2957	0.4157
	2	0.2200	0.7823	0.8126	0.5414	1.8469		0.7507	5.6595
	3	0.0785	0.9755	0.9787	0.1604	6.2335		0.9723	15.9226
	4	0.3528		0.3528			0.3528		
8	1	0.3058	0.1952	0.3628	1.6858	0.5932	0.2956		
	2	0.2529	0.5558	0.6106	0.8283	1.2073		0.4949	2.4532
	3	0.1732	0.8319	0.8497	0.4077	2.4531		0.8137	7.9784
	4	0.0608	0.9812	0.9831	0.1237	8.0819		0.9793	18.1669
9	1	0.2622	0.3421	0.4310	1.2166	0.8219		0.2197	0.3037
	2	0.2137	0.6430	0.6776	0.6308	1.5854		0.6064	4.4576
	3	0.1395	0.8663	0.8775	0.3180	3.1450		0.8550	10.0636
	4	0.0485	0.9852	0.9864	0.0982	10.1795		0.9840	20.1650
	5	0.2790		0.2790			0.2790		
10	1	0.2493	0.1564	0.2943	1.6942	0.5902	0.2382		
	2	0.2249	0.4541	0.5067	0.8876	1.1266		0.3945	1.9880
	3	0.1785	0.7073	0.7295	0.4894	2.0434		0.6844	6.4750
	4	0.1146	0.8913	0.8986	0.2551	3.9208		0.8839	11.9386
	5	0.0395	0.9880	0.9888	0.0799	12.5163		0.9872	21.9565

0.25 DB CHEBYSHEV DESIGN

Table 4.

Order	Section	Real Part	Imaginary Part	F_0	α	Q	−3 db Frequency	Peaking Frequency	Peaking Level
2	1	0.5621	0.7154	0.9098	1.2356	0.8093		0.4425	0.2502
3	1	0.3062	0.8712	0.9234	0.6632	1.5079		0.8156	4. 0734
	2	0.6124		0.6124			0.6124		
4	1	0.4501	0.3840	0.5916	1.5215	0.6572	0.5470		
	2	0.1865	0.9272	0.9458	0.3944	2.5356		0.9082	8.2538
5	1	0.3247	0.5892	0.6727	0.9653	1.0359		0.4917	1.4585
	2	0.1240	0.9533	0.9613	0.2580	3.8763		0.9452	11.8413
	3	0.4013		0.4013			0.4013		
6	1	0.3284	0.2593	0.4184	1.5697	0.6371	0.3730		
	2	0.2404	0.7083	0.7480	0.6428	1.5557		0.6663	4.3121
	3	0.0880	0.9675	0.9715	0.1811	5.5205		0.9635	14.8753
7	1	0.2652	0.4344	0.5090	1.0421	0.9596		0.3441	1. 0173
	2	0.1835	0.7828	0.8040	0.4565	2.1908		0.7610	7. 0443
	3	0.0655	0.9761	0.9783	0.1339	7.4679		0.9739	17.4835
	4	0.2944		0.2944			0.2944		
8	1	0.2543	0.1953	0.3206	1.5862	0.6304	0.2822		
	2	0.2156	0.5561	0.5964	0.7230	1.3832		0.5126	3.4258
	3	0.1441	0.8323	0.8447	0.3412	2.9309		0.8197	9.4683
	4	0.0506	0.9817	0.9830	0.1029	9.7173		0.9804	19.7624
9	1	0.2176	0.3423	0.4056	1.0730	0.9320		0.2642	0.8624
	2	0.1774	0.6433	0.6673	0.5317	1.8808		0.6184	5.8052
	3	0.1158	0.8667	0.8744	0.2649	3.7755		0.8589	11.6163
	4	0.0402	0.9856	0.9864	0.0815	12.2659		0.9848	21.7812
	5	0.2315		0.2315			0.2315		
10	1	0.2065	0.1565	0.2591	1.5940	0.6274	0.2267		
	2	0.1863	0.4543	0.4910	0.7588	1.3178		0.4143	3. 0721
	3	0.1478	0.7075	0.7228	0.4090	2.4451		0.6919	7.9515
	4	0.0949	0.8915	0.8965	0.2117	4.7236		0.8864	13.5344
	5	0.0327	0.9883	0.9888	0.0661	15.1199		0.9878	23.5957

0.5 DB CHEBYSHEV DESIGN

Table 5.

Order	Section	Real Part	Imaginary Part	F_0	α	Q	−3 db Frequency	Peaking Frequency	Peaking Level
2	1	0.5129	0.7225	1.2314	1.1577	0.8638		0.7072	0.5002
3	1	0.2683	0.8753	1.0688	0.5861	1.7061		0.9727	5.0301
	2	0.5366		0.6265			0.6265		
4	1	0.3872	0.3850	0.5969	1.4182	0.7051	0.5951		
	2	0.1605	0.9297	1.0313	0.3402	2.9391		1.0010	9.4918
5	1	0.2767	0.5902	0.6905	0.8490	1.1779		0.5522	2.2849
	2	0.1057	0.9550	1.0178	0.2200	4.5451		1.0054	13.2037
	3	0.3420		0.3623			0.3623		
6	1	0.2784	0.2596	0.3963	1.4627	0.6836	0.3827		
	2	0.2037	0.7091	0.7680	0.5522	1.8109		0.7071	5.5025
	3	0.0746	0.9687	1.0114	0.1536	6.5119		1.0055	16.2998
7	1	0.2241	0.4349	0.5040	0.9161	1. 0916		0.3839	1.7838
	2	0.1550	0.7836	0.8228	0.3881	2.5767		0.7912	8.3880
	3	0.0553	0.9771	1.0081	0.1130	8.8487		1.0049	18.9515
	4	0.2487		0.2562			0.2562		
8	1	0.2144	0.1955	0.2968	1.4779	0.6767	0.2835		
	2	0.1817	0.5565	0.5989	0.6208	1.6109		0.5381	4.5815
	3	0.1214	0.8328	0.8610	0.2885	3.4662		0.8429	10.8885
	4	0.0426	0.9824	1.0060	0.0867	11.5305		1.0041	21.2452
9	1	0.1831	0.3425	0.3954	0.9429	1.0605		0.2947	1.6023
	2	0.1493	0.6436	0.6727	0.4520	2.2126		0.6374	7.1258
	3	0.0974	0.8671	0.8884	0.2233	4.4779		0.8773	13.0759
	4	0.0338	0.9861	1.0046	0.0686	14.5829		1.0034	23.2820
	5	0.1949		0.1984			0.1984		
10	1	0.1736	0.1566	0.2338	1.4851	0.6734	0.2221		
	2	0.1566	0.4545	0.4807	0.6515	1.5349		0.4267	4.2087
	3	0.1243	0.7078	0.7186	0.3459	2.8907		0.6968	9.3520
	4	0.0798	0.8919	0.8955	0.1782	5.6107		0.8883	15.0149
	5	0.0275	0.9887	0.9891	0.0556	17.9833		0.9883	25.1008

1 DB CHEBYSHEV DESIGN

Table 6.

Order	Section	Real Part	Imaginary Part	F_0	α	Q	−3 db Frequency	Peaking Frequency	Peaking Level
2	1	0.4508	0.7351	0.8623	1.0456	0.9564		0.5806	0.9995
3	1	0.2257	0.8822	0.9106	0.4957	2.0173		0.8528	6.3708
	2	0.4513		0.4513			0.4513		
4	1	0.3199	0.3868	0.5019	1.2746	0.7845		0.2174	0.1557
	2	0.1325	0.9339	0.9433	0.2809	3.5594		0.9245	11.1142
5	1	0.2265	0.5918	0.6337	0.7149	1.3988		0.5467	3.5089
	2	0.0865	0.9575	0.9614	0.1800	5.5559		0.9536	14.9305
	3	0.2800		0.2800			0.2800		
6	1	0.2268	0.2601	0.3451	1.3144	0.7608		0.1273	0.0813
	2	0.1550	0.7106	0.7273	0.4262	2.3462		0.6935	7.6090
	3	0.0608	0.9707	0.9726	0.1249	8.0036		0.9688	18.0827
7	1	0.1819	0.4354	0.4719	0.7710	1.2971		0.3956	2.9579
	2	0.1259	0.7846	0.7946	0.3169	3.1558		0.7744	10.0927
	3	0.0449	0.9785	0.9795	0.0918	10.8982		0.9775	20.7563
	4	0.2019		0.2019			0.2019		
8	1	0.1737	0.1956	0.2616	1.3280	0.7530		0.0899	0.0611
	2	0.1473	0.5571	0.5762	0.5112	1.9560		0.5373	6.1210
	3	0.0984	0.8337	0.8395	0.2344	4.2657		0.8279	12.6599
	4	0.0346	0.9836	0.9842	0.0702	14.2391		0.9830	23.0750
9	1	0.1482	0.3427	0.3734	0.7938	1.2597		0.3090	2.7498
	2	0.1208	0.6442	0.6554	0.3686	2.7129		0.6328	8.8187
	3	0.0788	0.8679	0.8715	0.1809	5.5268		0.8643	14.8852
	4	0.0274	0.9869	0.9873	0.0555	18.0226		0.9865	25.1197
	5	0.1577		0.1577			0.1577		
10	1	0.1403	0.1567	0.2103	1.3341	0.7496		0.0698	0.0530
	2	0.1266	0.4548	0.4721	0.5363	1.8645		0.4368	5.7354
	3	0.1005	0.7084	0.7155	0.2809	3.5597		0.7012	11.1147
	4	0.0645	0.8926	0.8949	0.1441	6.9374		0.8903	16.8466
	5	0.0222	0.9895	0.9897	0.0449	22.2916		0.9893	26.9650

REFERENCES

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